Ladder Pricing – A New Form of Wholesale Price Discrimination

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Appendix: Notes on the Quantity Discounting Solution

Section 2 of the paper discusses the optimal (profit maximizing) solution for wholesale ladder pricing for the case where retailers have heterogeneous demands and (marginal) costs. The solution is compared with that obtained from optimal (profit maximizing) quantity discounting. Derivation of the optimal quantity discount pricing schedule was omitted from the paper. This was to save space, and because the technique used (the demand profile approach) is standard. That said, the derivation has some interest, because there are very few examples dealing with the bivariate distribution case in the literature, particularly where the optimal schedule prices some retailers out of the market.

This appendix uses the demand profile approach (e.g. Wilson [1993]) to solve for the optimal quantity discounting tariff (and associated wholesaler profit) for the case where demand is linear ($q=\theta-p$), and marginal cost α and the demand intercept θ have a bivariate uniform distribution on support $\left[0,1\right]\times\left[0,\theta_h\right]$. The approach is feasible because a change of variable allows the bivariate distribution problem to be effectively reduced to a univariate one (see below). The analysis involves straightforward (but rather extensive and tedious) algebra. This can be presented more compactly, but I have left intermediate steps in for the benefit of anyone who might (masochistically) wish to follow things in detail.

Let $W(q)$ be the payment made by a retailer when it purchases quantity *q*, whilst $w(q)$ is the marginal wholesale price (on the range of quantities chosen by retailers, it is assumed that $W(q)$ is continuous and differentiable, such that $w(q)=W'(q)$). A (θ,α) retailer's profit on purchasing quantity *q* is

$$\pi_r=(p-\alpha)q-W(q)=(\theta-q-\alpha)q-W(q)\quad, \tag{A.1}$$

and the first order necessary condition (*FONC*), for a retailer that participates, is

$$\partial\pi_r/\partial q=\theta-2q-\alpha-w(q)=0. \tag{A.2}$$

Define the variable

$$z:=w(q)+2q. \tag{A.3}$$

Then from (A.2)

$$\theta-\alpha=2q+w(q)=z\quad. \tag{A.4}$$

Notice that, for given $w(q)$, participating (θ,α) retailers with the same *z*-score choose the same *q*. Retailers who participate and buy *q* or more satisfy the condition

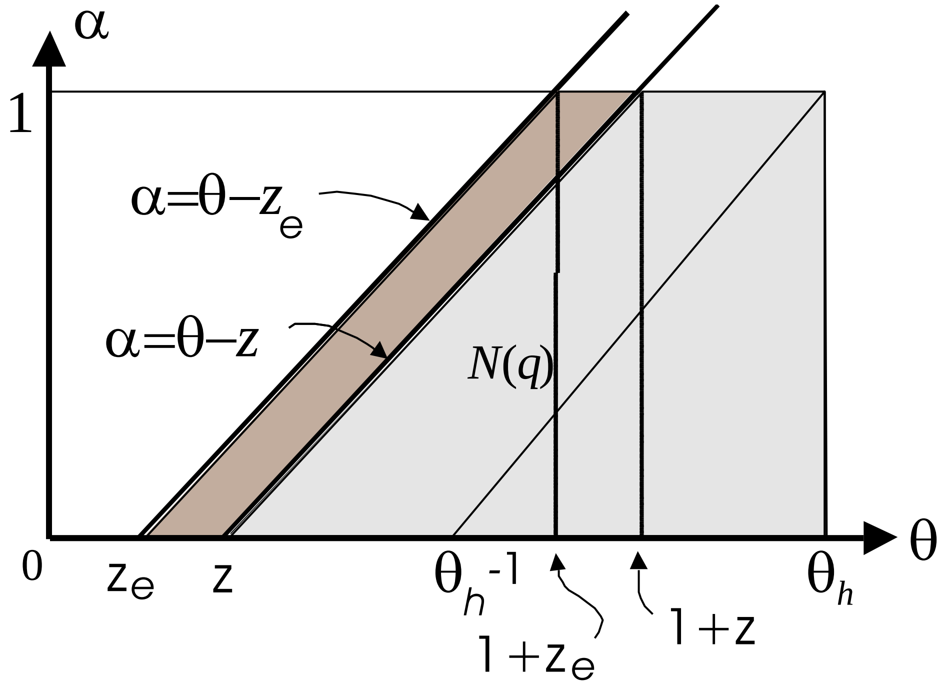
$$\theta-\alpha\geq z=2q+w(q)\quad. \tag{A.5}$$

In addition, there is the participation condition that the retailer profit π_r should be non-negative;

$$\pi_r = (\theta - q - \alpha)q - W(q) \geq 0 \quad . \tag{A.6}$$

Naturally, some retailers are excluded – they are priced out of the market. For example, all with $\theta < \alpha$ are certainly excluded as their costs are greater than their maximum willingness to pay (as θ is the upper limit of the demand curve). To the extent that they face a positive wholesale charge, exclusion will be greater. Thus there is a critical value for *z*, denoted z_e, such that all retailers with $\theta - \alpha \geq z_e$ participate (the optimal value for z_e is analyzed later). Given retailers with $\theta < \alpha$ are certainly excluded, clearly $z_e \geq 0$. Equally, if it were the case that $z_e > \theta_h$, this would mean zero participation and zero profit, so it must be that the optimal $z_e \in [0, \theta_h]$. The shape of the participation region is illustrated in figure A.1 below. [1]

Figure A.1



The participation region Ω is defined as

$$\Omega = \{(\theta, \alpha) : \theta \in [0, \theta_h], \alpha \in [0,1], \theta - \alpha \geq z_e\}$$

(the sum of the two shaded regions in figure A.1). The 'number' of retailers who choose at least *q* (for some arbitrary *q*) is denoted $N(q)$ (and the 'proportion' of retailers is simply $N(q)/\theta_h$ given the total area of the rectangle is θ_h). Both Ω and $N(q)$ are depicted as trapezoidal areas in figure A.1. However, notice that, if $z_e \geq \theta_h - 1$ then Ω is triangular, as is $N(q)$ if $z \geq \theta_h - 1$(this complicates the exposition somewhat). For a given choice of $w(q)$, *q* depends on *z* via (A.3), and

[1] Note that, as in Wilson's [1993, p. 157-8] original example, analysis based solely on *FONC*s is satisfactory – that is, for retailers who choose *q* >0, as per the *FONC* (A.2), the participation constraint (A.6) is indeed satisfied (it is straightforward to verify that this is the case once the optimal tariff has been found).

hence $N(q)$ is a function of z; the rate of change of $N(q)$ with respect to z will then be different depending on whether $z \geq \theta_h - 1$ or not. Specifically, from Figure A.1 the area $N(q)$ is given as

$$\begin{aligned} N(q) &= \tfrac{1}{2} + (\theta_h - 1 - z) = \theta_h - \tfrac{1}{2} - z && z \in [z_e, \max(z_e, \theta_h - 1)] \\ N(q) &= \tfrac{1}{2}(\theta_h - z)^2 && z \in [\max(z_e, \theta_h - 1), \theta_h]. \end{aligned} \tag{A.7}$$

The profit π_w earned by the wholesaler on those retailers who choose to buy the q^{th} unit is (using (A.3)):

$$\begin{aligned} \pi_w &= N(q)w(q) = w(q)\left(\theta_h - \tfrac{1}{2} - z\right) \\ &= w(q)\left(\theta_h - \tfrac{1}{2} - w(q) - 2q\right) && z \in [z_e, \max(z_e, \theta_h - 1)] \end{aligned}$$

$$\begin{aligned} \pi_w &= N(q)w(q) = w(q)\tfrac{1}{2}\left(\theta_h - z\right)^2 \\ &= w(q)\tfrac{1}{2}\left(\theta_h - w(q) - 2q\right)^2 && z \in [\max(z_e, \theta_h - 1), \theta_h]. \end{aligned} \tag{A.8}$$

The demand profile approach involves point-wise optimizing $w(q)$ for each value of q (this is followed by finding the optimal level of participation – see later). Thus the *FONC* is that

$$\begin{aligned} &\partial \pi_w / \partial w(q) = \left(\theta_h - \tfrac{1}{2} - w(q) - 2q\right) - w(q) = 0 \\ &\Rightarrow w^*(q) = \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right) - q && z \in [z_e, \max(z_e, \theta_h - 1)] \end{aligned}$$

$$\begin{aligned} &\partial \pi_w / \partial w(q) = \tfrac{1}{2}\left(\theta_h - w(q) - 2q\right)^2 - w(q)\left(\theta_h - w(q) - 2q\right) = 0 \\ &\Rightarrow \tfrac{1}{2}\left(\theta_h - w(q) - 2q\right) - w(q) = 0 \Rightarrow w^*(q) = \tfrac{1}{3}\theta_h - \tfrac{2}{3}q \\ &&& z \in [\max(z_e, \theta_h - 1), \theta_h]. \end{aligned} \tag{A.9}$$

This gives the optimal tariff for participating retailers defined for $z \in [z_e, \theta_h]$. Substituting for z using (A.3), and using the formula for $w^*(q)$, the ranges can be transposed into ranges for q; thus for $z \in [z_e, \max(z_e, \theta_h - 1)]$, given

$$z = 2q + w(q) = 2q + \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right) - q = q + \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right),$$
$$z_e = q_e + \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right),$$

so

$$\begin{aligned} &z \in [z_e, \max(z_e, \theta_h - 1)] \Rightarrow q + \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right) \in \left[q_e + \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right), \max(q_e + \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right), \theta_h - 1)\right] \\ &\Rightarrow q \in \left[q_e, \max\left(q_e, \theta_h - 1 - \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right)\right)\right] = \left[q_e, \max\left(q_e, \tfrac{1}{2}\theta_h - \tfrac{3}{4}\right)\right]. \end{aligned}$$

Similarly for $z \in [\max(z_e, \theta_h - 1), \theta_h]$; in this case,

$$z = 2q + w(q) = 2q + \tfrac{1}{3}\theta_h - \tfrac{2}{3}q = \tfrac{4}{3}q + \tfrac{1}{3}\theta_h,$$
$$z_e = \tfrac{4}{3}q_e + \tfrac{1}{3}\theta_h,$$

so

$$\begin{aligned} &z \in [\max(z_e, \theta_h - 1), \theta_h] \Rightarrow \tfrac{4}{3}q + \tfrac{1}{3}\theta_h \in \left[\max(\tfrac{4}{3}q_e + \tfrac{1}{3}\theta_h, \theta_h - 1), \theta_h\right] \\ &\Rightarrow q \in \left[\max(q_e, \tfrac{1}{2}\theta_h - \tfrac{3}{4}), \tfrac{1}{2}\theta_h\right], \end{aligned}$$

and so

$$w^*(q)=\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}\right)-q \qquad q\in\left[q_e, Max(q_e,\tfrac{1}{2}\theta_h-\tfrac{3}{4})\right]$$

$$w^*(q)=\tfrac{1}{3}\theta_h-\tfrac{2}{3}q \qquad q\in\left[Max(q_e,\tfrac{1}{2}\theta_h-\tfrac{3}{4}),\tfrac{1}{2}\theta_h\right] \tag{A.10}$$

is the optimal tariff. Notice this is continuous and piecewise linear. Notice also that the wholesale price schedule below q_e is undefined. Let $W_e=W(q_e)$ be the payment made by marginal retailers when they choose to buy q_e. Potentially, $W_e>0, q_e>0$, although in what follows it is proven that the optimal solutions actually require $W_e^*=0, q_e^*=0$. The associated outlay schedule is

$$W^*(q):=W^a(q)=W_e+\int_{q_e}^{q} w^*(t)dt = W_e+\int_{q_e}^{q}\left(\tfrac{1}{2}\theta_h-\tfrac{1}{4}-t\right)dt$$

$$=W_e+\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-q\right)q-\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-q_e\right)q_e \qquad q\in\left[q_e, Max(q_e,\tfrac{1}{2}\theta_h-\tfrac{3}{4})\right]$$

$$W^*(q):=W^b(q)=W_e+\int_{q_e}^{q} w^*(t)dt$$

$$=W_e+\int_{q_e}^{Max(q_e,\frac{1}{2}\theta_h-\frac{3}{4})}\left(\tfrac{1}{2}\theta_h-\tfrac{1}{4}-t\right)dt+\int_{Max(q_e,\frac{1}{2}\theta_h-\frac{3}{4})}^{q}\left(\tfrac{1}{3}\theta_h-\tfrac{2}{3}t\right)dt$$

$$q\in\left[Max(q_e,\tfrac{1}{2}\theta_h-\tfrac{3}{4}),\tfrac{1}{2}\theta_h\right] \tag{A.11}$$

(where it is convenient to label the wholesale revenue function as W^a, W^b for the two cases). Simplifying the latter,

$$W^b(q)=W_e+\left[\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-t\right)t\right]_{q_e}^{Max(q_e,\frac{1}{2}\theta_h-\frac{3}{4})}+\tfrac{1}{3}\left[\left(\theta_h-t\right)t\right]_{Max(q_e,\frac{1}{2}\theta_h-\frac{3}{4})}^{q}$$

$$q\in\left[Max(q_e,\tfrac{1}{2}\theta_h-\tfrac{3}{4}),\tfrac{1}{2}\theta_h\right], \tag{A.12}$$

so if $q_e\le\tfrac{1}{2}\theta_h-\tfrac{3}{4}\;(z_e\le\theta_h-1)$,

$$W^b(q)=W_e+\left[\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-t\right)t\right]_{q_e}^{\frac{1}{2}\theta_h-\frac{3}{4}}+\tfrac{1}{3}\left[\left(\theta_h-t\right)t\right]_{\frac{1}{2}\theta_h-\frac{3}{4}}^{q}$$

$$=W_e+\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-\left(\tfrac{1}{2}\theta_h-\tfrac{3}{4}\right)\right)\left(\tfrac{1}{2}\theta_h-\tfrac{3}{4}\right)-\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-q_e\right)q_e$$

$$+\tfrac{1}{3}\left[\left(\theta_h-q\right)q-\left(\theta_h-\left(\tfrac{1}{2}\theta_h-\tfrac{3}{4}\right)\right)\left(\tfrac{1}{2}\theta_h-\tfrac{3}{4}\right)\right]$$

$$=W_e+\left(\tfrac{1}{24}\theta_h^2-\tfrac{1}{8}\theta_h+\tfrac{3}{32}\right)-\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-q_e\right)q_e+\tfrac{1}{3}\left(\theta_h-q\right)q,$$

whilst if $q_e>\tfrac{1}{2}\theta_h-\tfrac{3}{4}\;(z_e>\theta_h-1)$

$$W^b(q)=W_e+\tfrac{1}{3}\left[\left(\theta_h-t\right)t\right]_{q_e}^{q}=W_e+\tfrac{1}{3}\left(\left(\theta_h-q\right)q-\left(\theta_h-q_e\right)q_e\right). \tag{A.13}$$

The optimal choice of $q(\theta,\alpha)$ is given from (A.4) and (A.10);

$$2q+w^*(q)=\theta-\alpha, \tag{A.14}$$

so that

$$2q+w^*(q)=\theta-\alpha \Rightarrow 2q+\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}\right)-q=\theta-\alpha$$

$$\Rightarrow q(\theta,\alpha)=\theta-\alpha-\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}\right)=z-\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}\right) \qquad z\in\left[z_e,\max(z_e,\theta_h-1)\right]$$

$$2q+w^*(q)=\theta-\alpha \Rightarrow 2q+\tfrac{1}{3}\theta_h-\tfrac{2}{3}q=\theta-\alpha$$

$$\Rightarrow q(\theta,\alpha)=\tfrac{3}{4}\left(\theta-\alpha\right)-\tfrac{1}{4}\theta_h=\tfrac{3}{4}z-\tfrac{1}{4}\theta_h \qquad z\in\left[\max(z_e,\theta_h-1),\theta_h\right]. \tag{A.15}$$

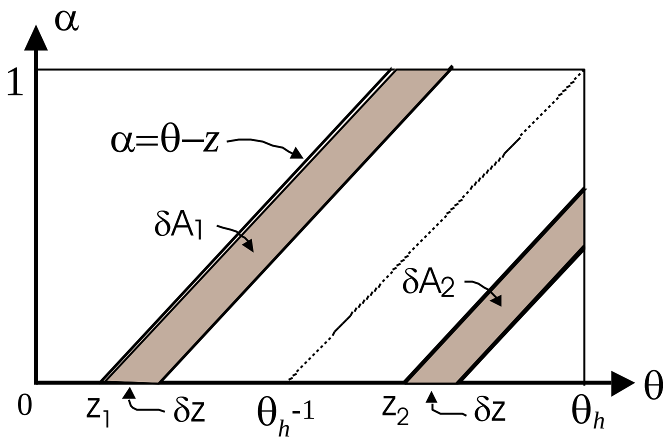
Note that this means the payment *W* can be viewed as a function of *q* or of *z* – by substituting (A.15) into (A.11)/(A.13) (it is convenient, with a slight abuse of notation, to write $W(q)$ or $W(z)$ in what follows to indicate this dependence).

Given there is uniform density of $1/\theta_h$ on $[0,1]\times[0,\theta_h]$, overall profit earned by the wholesaler, Π_W can be written as

$$\Pi_W = \iint_\Omega W\left(q(\theta,\alpha)\right)(1/\theta_h)d\theta d\alpha$$
$$= (1/\theta_h)\left(\int_{z_e}^{Max(z_e,\theta_h-1)} W^a\left(z\right)dz + \int_{Max(z_e,\theta_h-1)}^{\theta_h}\left(\theta_h - z\right)W^b\left(z\right)dz\right) \quad \text{(A.16)}$$

where the simplification from double integral to single integral can be explained diagrammatically (see figure A.2).

Figure A.2



First note that the strip of area δA_1 in figure A.2 is equal to δz (area of a parallelogram is simply its vertical height times its base). For a given value of *z*, the same quantity *q* is taken, and the same payment *W* is made, hence the value of the integral is given as the integral of $W(z)dz$ for $z\in\left[0,\theta_h-1\right]$ as in the line 2 first integral in (A.16). When *z* enters the range $\left[\theta_{h-1},\theta_h\right]$, the area is as depicted by δA_2 at z_2 in figure A.2; the area is now given as the difference in the two right angled triangles with sides of length $\theta_h - z$ and $\theta_h - z - \delta z$; thus

$$\delta A = \tfrac{1}{2}\left(\theta_h - z\right)^2 - \tfrac{1}{2}\left(\theta_h - z - \delta z\right)^2$$
$$= \tfrac{1}{2}\left(\theta_h - z\right)^2 - \tfrac{1}{2}\left(\theta_h - z\right)^2 + \left(\theta_h - z\right)\delta z - \tfrac{1}{2}\delta z^2 = \left(\theta_h - z\right)\delta z - \tfrac{1}{2}\delta z^2.$$

Hence value of line 2 second integral in (A.16) on the interval $\left[\theta_{h-1},\theta_h\right]$ is given by the integral of $W(z)\left(\theta_h - z\right)dz$.

Participation analysis:

The question arises as to whether, for a given choice of parameter value for θ_h, whether $z_e < \theta_h - 1$ or not (equivalently, whether q_e is greater or less than $\frac{1}{2}\theta_h - \frac{3}{4}$), since this affects the formula for W.

<u>Case (a)</u> – if $q_e \le \frac{1}{2}\theta_h - \frac{3}{4}$ ($\Leftrightarrow z_e \le \theta_h - 1$):

Marginal retailers have zero profitability:

$$\begin{aligned}\pi_r &= (p_e-\alpha_e)q_e - W_e = (\theta_e - q_e - \alpha_e)q_e - W_e \\ &= (z_e - q_e)q_e - W_e = 0,\end{aligned} \tag{A.17}$$

and the retailer *FONC* holds:

$$\begin{aligned}&z_e = 2q_e + w^*(q_e) = 2q_e + \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right) - q_e = q_e + \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right) \\ &\Rightarrow z_e - q_e = \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right).\end{aligned} \tag{A.18}$$

Notice there is three choice variables, z_e, q_e, W_e and two equations. Choosing z_e thus determines both q_e, W_e. Specifically,

$$q_e = z_e - \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right), \tag{A.19}$$

and

$$\begin{aligned}W_e &= (z_e - q_e)q_e = \left(z_e - \left(z_e - \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right)\right)\right)\left(z_e - \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right)\right) \\ &= \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right)\left(z_e - \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2}\right)\right).\end{aligned} \tag{A.20}$$

Since in this case $z_e \le \theta_h - 1$, so $Max(z_e, \theta_h - 1) = \theta_h - 1$, the overall profit, (A.16), can be written as

$$\begin{aligned}\theta_h \Pi_W &= \int_{z_e}^{\theta_h - 1} W^a(z)dz + \int_{\theta_h - 1}^{\theta_h} (\theta_h - z) W^b(z)dz \\ &= \int_{z_e}^{\theta_h - 1}\left(W_e + \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2} - q\right)q - \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2} - q_e\right)q_e\right)dz \\ &+ \int_{\theta_h - 1}^{\theta_h}(\theta_h - z)\left(W_e + \left(\tfrac{1}{24}\theta_h^2 - \tfrac{1}{8}\theta_h + \tfrac{3}{32}\right) - \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2} - q_e\right)q_e + \tfrac{1}{3}\left(\theta_h - q\right)q\right)dz.\end{aligned} \tag{A.21}$$

Now

$$\int_{\theta_h - 1}^{\theta_h}(\theta_h - z)dz = \tfrac{1}{2} \tag{A.22}$$

(the area of the right angled triangle with unit length base and height), so

$$\begin{aligned}&\int_{\theta_h - 1}^{\theta_h}(\theta_h - z)\left(W_e + \left(\tfrac{1}{24}\theta_h^2 - \tfrac{1}{8}\theta_h + \tfrac{3}{32}\right) - \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2} - q_e\right)q_e\right)dz \\ &= \tfrac{1}{2}\left(W_e + \left(\tfrac{1}{24}\theta_h^2 - \tfrac{1}{8}\theta_h + \tfrac{3}{32}\right) - \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2} - q_e\right)q_e\right).\end{aligned} \tag{A.23}$$

Also

$$\int_{z_e}^{\theta_h - 1}\left(W_e - \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2} - q_e\right)q_e\right)dz = \left(W_e - \tfrac{1}{2}\left(\theta_h - \tfrac{1}{2} - q_e\right)q_e\right)\left(\theta_h - 1 - z_e\right)\ , \tag{A.24}$$

and

$$\begin{aligned}
&\int_{z_e}^{\theta_h-1}\left(\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-q\right)q\right)dz+\int_{\theta_h-1}^{\theta_h}\left(\theta_h-z\right)\left(\tfrac{1}{3}\left(\theta_h-q\right)q\right)dz\\
&=\int_{z_e}^{\theta_h-1}\left(\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-\left(z-\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}\right)\right)\right)\left(z-\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}\right)\right)\right)dz\\
&+\int_{\theta_h-1}^{\theta_h}\left(\theta_h-z\right)\left(\tfrac{1}{3}\left(\theta_h-\left(\tfrac{3}{4}z-\tfrac{1}{4}\theta_h\right)\right)\left(\tfrac{3}{4}z-\tfrac{1}{4}\theta_h\right)\right)dz\\
&=\int_{z_e}^{\theta_h-1}\left(6\theta_h-3-4z\right)\left(4z-2\theta_h+1\right)dz+\tfrac{1}{48}\int_{\theta_h-1}^{\theta_h}\left(\theta_h-z\right)\left(4\theta_h-3z+\theta_h\right)\left(3z-\theta_h\right)dz,
\end{aligned} \tag{A.25}$$

where

$$\left(6\theta_h-3-4z\right)\left(4z-2\theta_h+1\right)=\left(-12\theta_h^2+12\theta_h-3+32z\theta_h-16z-16z^2\right) \tag{A.26}$$

and

$$\left(\theta_h-z\right)\left(4\theta_h-3z+\theta_h\right)\left(3z-\theta_h\right)=-5\theta_h^3+23\theta_h^2z-27\theta_hz^2+9z^3. \tag{A.27}$$

Putting this together,

$$\begin{aligned}
\theta_h\Pi_W&=\int_{z_e}^{\theta_h-1}\left(W_e+\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-q\right)q-\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-q_e\right)q_e\right)dz\\
&+\int_{\theta_h-1}^{\theta_h}\left(\theta_h-z\right)\left(W_e+\left(\tfrac{1}{24}\theta_h^2-\tfrac{1}{8}\theta_h+\tfrac{3}{32}\right)-\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-q_e\right)q_e+\tfrac{1}{3}\left(\theta_h-q\right)q\right)dz\\
&=\left(W_e-\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-q_e\right)q_e\right)\left(\theta_h-1-z_e\right)+\tfrac{1}{2}\left(W_e-\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}-q_e\right)q_e+\left(\tfrac{1}{24}\theta_h^2-\tfrac{1}{8}\theta_h+\tfrac{3}{32}\right)\right)\\
&+\tfrac{1}{32}\int_{z_e}^{\theta_h-1}\left(6\theta_h-3-4z\right)\left(4z-2\theta_h+1\right)dz+\tfrac{1}{48}\int_{\theta_h-1}^{\theta_h}\left(\theta_h-z\right)\left(4\theta_h-3z+\theta_h\right)\left(3z-\theta_h\right)dz.
\end{aligned} \tag{A.28}$$

Substituting for W_e, expanding, and performing the integrations gives

$$\begin{aligned}
\theta_h\Pi_W&=\tfrac{1}{2}\left(z_e-\tfrac{1}{2}\theta_h+\tfrac{1}{4}\right)^2\left(\theta_h-z_e-\tfrac{1}{2}\right)+\tfrac{1}{2}\left(\tfrac{1}{24}\theta_h^2-\tfrac{1}{8}\theta_h+\tfrac{3}{32}\right)\\
&+\tfrac{1}{32}\left(\left(-12\theta_h^2+12\theta_h-3\right)\left(\theta_h-1-z_e\right)+8\left(2\theta_h-1\right)\left(\left(\theta_h-1\right)^2-z_e^2\right)-\tfrac{16}{3}\left(\left(\theta_h-1\right)^3-z_e^3\right)\right)\\
&+\tfrac{1}{48}\left(-5\theta_h^3+\tfrac{23}{2}\theta_h^2\left(\theta_h^2-\left(\theta_h-1\right)^2\right)-\tfrac{27}{3}\theta_h\left(\theta_h^3-\left(\theta_h-1\right)^3\right)+\tfrac{9}{4}\left(\theta_h^4-\left(\theta_h-1\right)^4\right)\right).
\end{aligned} \tag{A.29}$$

The optimal choice of z_e can now be found via the *FONC* that

$$\begin{aligned}
\partial\left(\theta_h\Pi_W\right)/\partial z_e&=-\tfrac{1}{2}\left(z_e-\tfrac{1}{2}\theta_h+\tfrac{1}{4}\right)^2+\left(z_e-\tfrac{1}{2}\theta_h+\tfrac{1}{4}\right)\left(\theta_h-z_e-\tfrac{1}{2}\right)\\
&+\tfrac{1}{32}\left(-12\left(-\theta_h^2+\theta_h-\tfrac{1}{4}\right)-16\left(2\theta_h-1\right)z_e+16z_e^2\right)=0.
\end{aligned} \tag{A.30}$$

This can be simplified to get

$$z_e{*}=\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}\right). \tag{A.31}$$

Substituting this back into (A.29) gives the optimal solution. Notice also that, from (A.19) and (A.31), $q_e{*}=0$ and hence from (A.20), $W_e{*}=0$. That is the wholesale tariff is defined for all $q\geq 0$.
Finally, note that this case applies when

$$z_e\leq\theta_h-1\Rightarrow\tfrac{1}{2}\left(\theta_h-\tfrac{1}{2}\right)\leq\theta_h-1\Rightarrow\theta_h\geq\tfrac{3}{2}$$

(and case (b) below will be shown to apply when $\theta_h<\tfrac{3}{2}$).

<u>Case (b)</u> – if $q_e>\tfrac{1}{2}\theta_h-\tfrac{3}{4}$: or $z_e>\theta_h-1$

As in (A.17), marginal retailers have zero profitability:

$$\begin{aligned}\pi_r &= (p_e-\alpha_e)q_e - W_e = (\theta_e - q_e - \alpha_e)q_e - W_e \\ &= (z_e - q_e)q_e - W_e = 0 \Rightarrow W_e = (z_e-q_e)q_e.\end{aligned} \tag{A.32}$$

and the retailer *FONC* holds, but in this case $w^*(q) = \frac{1}{3}\theta_h - \frac{2}{3}q$, so

$$\begin{aligned}& z_e = 2q_e + w^*(q_e) = 2q_e + \tfrac{1}{3}\theta_h - \tfrac{2}{3}q_e = \tfrac{4}{3}q_e + \tfrac{1}{3}\theta_h \\ & \Rightarrow z_e - q_e = \tfrac{1}{3}\left(q_e+\theta_h\right), q_e = \tfrac{3}{4}z_e - \tfrac{1}{4}\theta_h.\end{aligned} \tag{A.33}$$

As before there are three choice variables, z_e, q_e, W_e and two equations relating these variables. Choosing z_e thus determines both q_e, W_e. From (A.32),(A.33),

$$W_e = (z_e-q_e)q_e = (z_e - \tfrac{3}{4}z_e + \tfrac{1}{4}\theta_h)\left(\tfrac{3}{4}z_e - \tfrac{1}{4}\theta_h\right) = \tfrac{1}{16}(z_e+\theta_h)\left(3z_e - \theta_h\right) \tag{A.34}$$

Since in this case $z_e > \theta_h - 1$, so $Max(z_e, \theta_h - 1) = z_e$, the overall profit, (A.16), can be written as

$$\theta_h \Pi_W = \int_{z_e}^{\theta_h} \left(\theta_h - z\right) W^b\left(z\right) dz = \int_{z_e}^{\theta_h}\left(\theta_h - z\right)\left(W_e + \tfrac{1}{3}\left(\left(\theta_h - q\right)q - \left(\theta_h - q_e\right)q_e\right)\right)dz. \tag{A.35}$$

Note the first integral in the RHS of (A.16) is now zero. Note also that $W^b(q)$ is given by (A.13) as $W^b(q) = W_e + \frac{1}{3}\left(\left(\theta_h - q\right)q - \left(\theta_h - q_e\right)q_e\right)$ where $q = \frac{3}{4}z - \frac{1}{4}\theta_h$ and $q_e = \frac{3}{4}z_e - \frac{1}{4}\theta_h$ from (A.15). Hence

$$\begin{aligned}\theta_h\Pi_W &= \int_{z_e}^{\theta_h}\left(\theta_h - z\right)\left(W_e + \tfrac{1}{3}\left(\left(\theta_h - \left(\tfrac{3}{4}z - \tfrac{1}{4}\theta_h\right)\right)\left(\tfrac{3}{4}z - \tfrac{1}{4}\theta_h\right) - \left(\theta_h - \left(\tfrac{3}{4}z_e - \tfrac{1}{4}\theta_h\right)\right)\left(\tfrac{3}{4}z_e - \tfrac{1}{4}\theta_h\right)\right)\right)dz \\ &= \int_{z_e}^{\theta_h}\left(\theta_h - z\right)\left(\tfrac{1}{16}(z_e+\theta_h)\left(3z_e - \theta_h\right) + \tfrac{1}{3}\left(\tfrac{1}{16}\left(5\theta_h - 3z\right)\left(3z - \theta_h\right) - \tfrac{1}{16}\left(5\theta_h - 3z_e\right)\left(3z_e - \theta_h\right)\right)\right)dz.\end{aligned} \tag{A.36}$$

Expanding and integrating, this eventually gives

$$\theta_h\Pi_W = \tfrac{1}{48}\left(\alpha_0 z + \tfrac{1}{2}\alpha_1 z^2 + \tfrac{1}{3}\alpha_2 z^3 + \tfrac{9}{4}z^4\right)_{z_e}^{\theta_h}, \tag{A.37}$$

where

$$\begin{aligned}\alpha_0 &= -3\theta_h^{\,3} - 12\theta_h^{\,2} z_e + 18\theta_h z_e^{\,2}, \\ \alpha_1 &= 21\theta_h^{\,2} + 12\theta_h z_e - 18 z_e^{\,2}, \\ \alpha_2 &= -27\theta_h.\end{aligned} \tag{A.38}$$

Optimal z_e is found via the *FONC* as in case (a); thus

$$\partial\theta_h\Pi_W / \partial z_e = -\tfrac{1}{48}\begin{pmatrix}\left(-3\theta_h^{\,3} - 12\theta_h^{\,2}z_e + 18\theta_h z_e^{\,2}\right) + \left(18\theta_h^{\,2} + 3\theta_h^{\,2} + 12\theta_h z_e - 18 z_e^{\,2}\right)z_e \\ + \left(-9\theta_h - 18\theta_h\right)z_e^{\,2} + 9z_e^{\,3}\end{pmatrix}$$

$$+\tfrac{1}{48}\int_{z_e}^{\theta_h}\left(\left(-12\theta_h^{\,2} + 36\theta_h z_e\right) + \left(12\theta_h - 36 z_e\right)z\right)dz = 0,$$

which can eventually be reduced to

$$\theta_h^{\,3} - 7\theta_h^{\,2}z_e + 11\theta_h z_e^{\,2} + 3z_e^{\,3} = 0. \tag{A.39}$$

Writing $x = z_e/\theta_h$, (A.39) can be factorized to get

$$3(x - \tfrac{1}{3})(x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) = 0. \tag{A.40}$$

The solutions are thus $z_e/\theta_h = \frac{1}{3}$, or $2+\sqrt{3}$ or $2-\sqrt{3}$. The second solution is greater than 1, which violates the requirement that $z_e \in \left[0, \theta_h\right]$), and the third solution violates non-negativity (that $q_e \geq 0$; $q_e = \frac{3}{4}z_e - \frac{1}{4}\theta_h = \frac{3}{4}\left(2-\sqrt{3}\right)\theta_h - \frac{1}{4}\theta_h = \frac{1}{4}\left(5 - 3\sqrt{3}\right)\theta_h < 0$). So the solution is $z_e^* = \frac{1}{3}\theta_h$. This solution also implies $q_e^* = \frac{3}{4}z_e^* - \frac{1}{4}\theta_h = 0$ and $W_e^* = 0$.

Notice that $z_e > \theta_h - 1 \Rightarrow \frac{1}{3}\theta_h > \theta_h - 1 \Rightarrow \theta_h < \frac{3}{2}$; that is, this case applies when $\theta_h < \frac{3}{2}$.

Profit is then given by (A.37) evaluated at z_e*. This completes the analysis; equations (A.29) and (A.37) are used (in conjunction with the respective formula for z_e*) in calculating profitability under optimal quantity discounting and hence in the construction of the comparison table in the main paper.